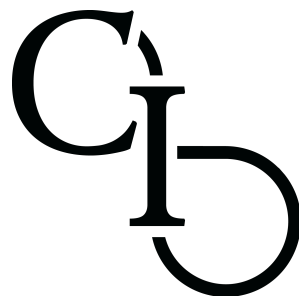


History of Ideas

Module 2

Ancient Greeks: The Pythagoreans

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CONJECTURE UNIVERSITY

Ancient Greeks: The Pythagoreans

I swear by the discoverer of the Tetraktys,

Which is the spring of all our wisdom,

The perennial root of Nature's fount.

-The Pythagorean Oath

“All Things Are Number”

The ancient Greeks did not appreciate our modern boundaries between science, philosophy, mathematics, and spirituality. For example, the Pythagoreans seamlessly wove together serious mathematical investigation with religious reverence for the beauty and power of numbers. Given the time they spent pursuing the magical and metaphysical meaning of numbers, our modern sensibilities would not recognize the Pythagoreans as strict mathematicians. Still, they made objective progress not only in the *subject* of mathematics but in understanding the *relationship* between mathematics and the rest of Nature.

Pythagoras (~572 - 497 BC) himself led this fusionistic charge, simultaneously working as a mathematician, philosopher, and spiritual leader. We do not know which of the Pythagorean school's genuine contributions were his own. He left no writings, and all of his acolytes attributed their discoveries to Pythagoras himself—possibly because they saw him as a religious leader.

As Mario Livio writes in *Is God a Mathematician?*, “To the Pythagoreans, numbers were both living entities and universal principles, permeating everything from the heavens to human ethics. In other words, numbers had two distinct, complementary aspects. On one hand, they had a tangible physical existence; on the other, they were abstract prescriptions on which everything was founded. For instance, the monad (the number 1) was understood both as the generator of all other numbers, an entity as real as water, air, and fire that participated in the structure of the physical world, and as an idea—the metaphysical unity at the source of all creation.”

The supposedly divine Tetraktys exemplifies the Pythagoreans' wedding of mysticism and respect for what we would consider genuine mathematics. This object is a special case of their more general way of representing numbers—they would create symmetrical (equilateral) triangles made out of discrete pebbles. For example, the number '1' would be represented by a single pebble, the number '3' would be represented by a total of six pebbles arranged as a row of one pebble stacked on a row of two pebbles stacked on a row of three pebbles. The Tetraktys is simply the particular arrangement of pebbles that represents the number '4' in accordance with their schema (stacked rows of one, two, three, and four pebbles from top to bottom). Because the total number of pebbles that make up the Tetraktys is ten, the Pythagoreans felt that the object represented nothing short of perfection.

In typical Pythagorean fashion, they went further in their mystical applications of the Tetraktys. For example, Pythagoreans are rightly credited with having discovered that “dividing a [musical instrument's] string by simple consecutive integers produces harmonious and consonant intervals.” In particular, they realized that plucking two strings at the same time whose lengths are in a 3:4 ratio with one another creates a pleasurable tune called the 'perfect fourth.' Naturally, they associated this seemingly magical harmony between space, music, joy, and number with the Tetraktys.

The Pythagoreans similarly applied their unique blend of rigorous mathematical inquiry and mysticism to the heavens. They were some of the first thinkers to assert that the Earth was a sphere, though at least one of their reasons was simply that spheres were 'superior' to other shapes. Perhaps more impressive, they may have been the first school to ever notice that the Sun, the Moon, and other planets move across the sky independently of the supposedly fixed background of stars. The Pythagoreans' love of the constellations only reinforced their divine view of number and geometry: each was defined by its *number* of constituent stars and *shape* that the stars collectively formed.

The Pythagoreans did not confine the supposed divinity of numbers to inanimate objects but rather thought that they infused the world of people just as much. For example, they thought that odd numbers were masculine, even numbers feminine. They also associated the concept of unity with the number one (understandably) and the concept of justice with the number four (less understandably).

Whether in the small, manmade musical instruments, in the titanic, celestial bodies, or in the abstract world of human ideas, Pythagoreans took for granted the effectiveness of mathematics while insisting that said effectiveness revealed a spiritual dimension of the world. Theirs was an inseparable mess of sound application and ad hoc storytelling.

The Dawn of Pure Mathematics and Proofs

Despite their cloak of mysticism, the Pythagoreans were the first to think seriously about how mathematics relates to both the physical world and human thought. They were also among the first thinkers to (attempt to) explain the order of the universe by harnessing mathematics.

Before the Pythagoreans, peoples like the Babylonians and the Egyptians saw mathematics as a purely practical tool with which they might improve their food production and other such pressing issues. For example, the Babylonians applied fairly sophisticated mathematics to problems in administration, surveying, construction, and engineering. The Pythagoreans, though, founded the concept of *pure* mathematics—the study of mathematical objects for their own sake. Famously, it was they who discovered the Pythagorean theorem—that the sum of the squares of a right triangle’s shorter sides equals the square of the length of its hypotenuse. The Egyptians had some familiarity with the theorem, but only the Pythagoreans explicitly recognized it as universal for all right triangles. They also discovered irrational numbers—numbers that cannot be expressed as the ratio between two integers (see below).

The Pythagoreans’ most important mathematical innovation was that of a mathematical *proof*. For the Pythagoreans, a piece of mathematics should have a logical procedure behind it that takes as input some initial postulates or axioms and outputs a mathematical theorem or some other surprising piece of mathematical knowledge. Just as the predecessors to the Pythagoreans chose applied mathematics over pure mathematics, so too were they satisfied merely that a mathematical formula worked for their purposes. The Pythagoreans, on the other hand, demanded to know *why* a formula worked—and they regarded the proof process as the definitive means of discovery. Their emphasis on the importance of proofs would lay the groundwork for Euclid, another ancient Greek thinker whom we will meet later in this course.

Irrational Failure

While they made great strides in mathematics, the Pythagorean school proved too cultish to survive as a long-lasting tradition of open-ended criticism (see our Civilization course). Its own discovery of irrational numbers violently contradicted their religiously held view that all of the natural world could be captured by whole numbers and their ratios. This was “a staggering blow,” writes Susan Wise Bauer in *The Story of Western Science*. Citing a commentator who lived not long after Pythagoras, she relays that “the man who first made public the theory of irrationals perished in a shipwreck in order that the inexpressible and unimaginable should ever remain veiled.” While surely apocryphal, this entire episode exposes the Pythagorean school as too dogmatic in its mysticism to make continual progress in either pure mathematics or the application of mathematical tools to scientific questions.

Apparently, the decades that separated Thales from Pythagoras were not enough time for the former’s tradition of criticism to inspire the latter. Before such a tradition takes hold, it is not obvious that openness to alternative theories, willingness to admit one might be wrong, and acceptance that even our most cherished ideas may contain mistakes are not only beneficial but essential for infinite progress. The Pythagoreans failed to adopt this attitude in time, and irrational numbers ended their irrational tradition.

